MathConception

2024

S4-6

Question Booklet

問題簿

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.
2. If the information printed on your answer sheet is not correct, please inform the invigilator immediately.
3. Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.
4. Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.
5. Rough-work sheets provided will be collected at the end of the contest but they will not be marked.
6. Diagrams in this question booklet are not necessarily drawn to scale.

限時：1 小時
不允許使用計算機。

比賽須知：

1. 未宣布開始前，切勿翻閱此問題簿。

2. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。

3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。

4. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。

5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。

6. 此問題簿的附圖不一定依比例繪成。

1) Find the negative constant k such that the straight lines $kx + 3y - 4 = 0$ and [3%]
$12x - \dfrac{y}{k} + 2 = 0$ are perpendicular to each other.

求使直線 $kx + 3y - 4 = 0$ 和直線 $12x - \dfrac{y}{k} + 2 = 0$ 互相垂直的負常數 k。

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2) Given that $g\left(\dfrac{k-x}{k+x}\right) = x$, for some constant k. Express $g(x)$ in terms of x. [3.1%]

已知對於某常數 k，$g\left(\dfrac{k-x}{k+x}\right) = x$。以 x 表示 $g(x)$。

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3) If the remainder of dividing $A(x)$ by $1 - 2x$ is R, then the remainder of [3.2%]
dividing $A(x)$ by $x - \dfrac{1}{2}$ is _____?_____ .

若當 $A(x)$ 除以 $1 - 2x$ 時的餘數是 R，
則當 $A(x)$ 除以 $x - \dfrac{1}{2}$ 時的餘數是 _____?_____ 。

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4) Given that $g(y) = ay^{15} + by^3 - c^{21}y^9$ and $g(d) = \log_3 29$, where a, b, c and d are [3.3%]
constants, find the value of $g(-d)$.

已知 $g(y) = ay^{15} + by^3 - c^{21}y^9$ 及 $g(d) = \log_3 29$，其中 a、b、c 和 d 都是常
數，求 $g(-d)$ 的值。

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5) Express the following in form of $a + bi$, where a and b are real numbers and i [3.4%]
is the imaginary number defined by $i^2 = -1$.

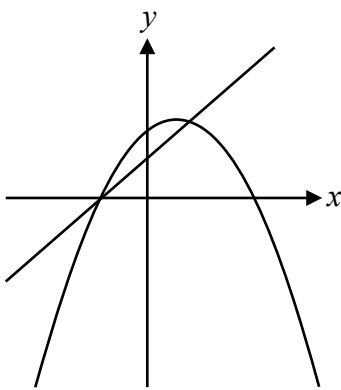以 $a + bi$ 的形式表示下列算式，其中 a 和 b 為實數，而 i 為虛數，其定
義為 $i^2 = -1$。

$$\frac{(-i)^9 - (-i)^{-414} - (-i^3) \div (-i)^{2024}}{-i^2 \times (-i)^{-3} - (-i)^8}$$

6) Observe the following figures. Which of the following figures could be [3.5%] representing the graphs of $f(x) = ax + b$ and $g(x) = ax^2 + bx$?
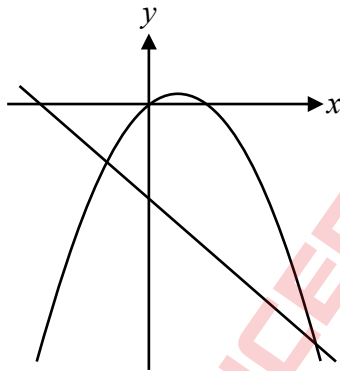(*a* and *b* are non-zero real numbers.)

觀察下列各圖。何者有可能是在表示 $f(x) = ax + b$ 和 $g(x) = ax^2 + bx$ 的圖像？（*a* 和 *b* 為非零實數。）

A.

B.

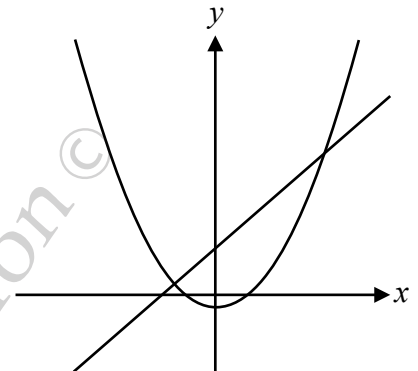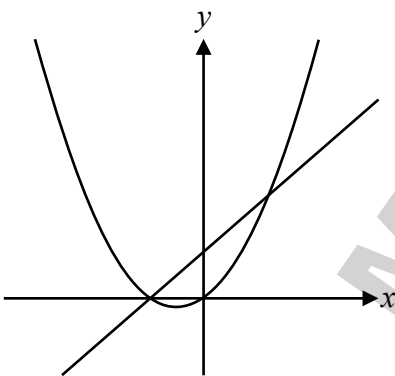C.

D.

E.
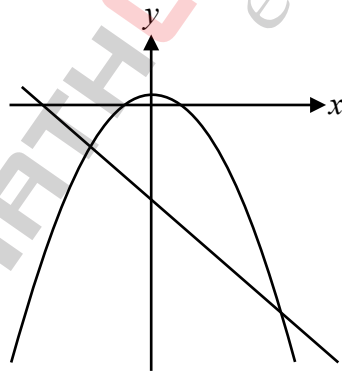
F.

G.

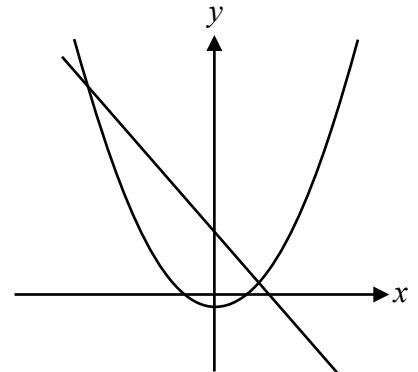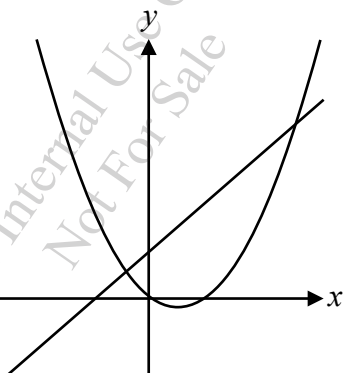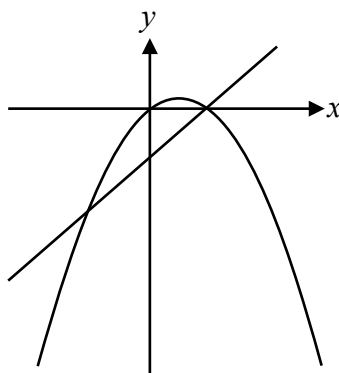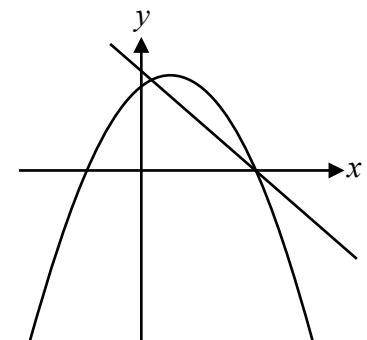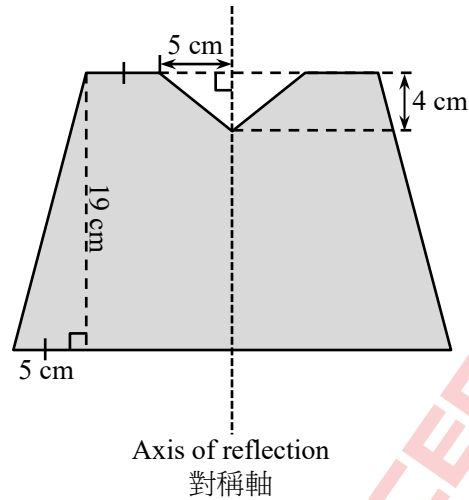H.

K.

7) In the figure, all the lengths are measured correct to the nearest cm. The [3.6%]
upper limit of the area of the shaded region is __?__ cm².

在圖中，所有長度量度值均準確至最接近的 cm。陰影部分面積的上限
是 __?__ cm²。

Axis of reflection
對稱軸

8) Given an unfair six-sided dice, the probabilities of obtaining 1, 2, 3, 4, 5 and [3.7%]
6 are in the ratio $15 : 6 : 8 : 12 : 10 : 9$. If two of such dice are rolled, find the
probability that the difference between the numbers on the two dice is 3.

某枚不勻稱的六面骰子擲得 1、2、3、4、5 和 6 的概率之比為
$15 : 6 : 8 : 12 : 10 : 9$。若投擲兩枚這樣的骰子，求兩枚骰子擲得的點數
之差為 3 的概率。

9) $\dfrac{\sin(-45°)}{\cos(-45°)} - \dfrac{\sin(44°)}{\cos(-46°)} + \dfrac{\sin(-43°)}{\cos(-47°)} - \dfrac{\sin(42°)}{\cos(-48°)} + \cdots + \dfrac{\sin(-1°)}{\cos(-89°)} = $ _____ ? _____ [4.8%]

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10) $x^2 - 4x + 5 = 0$ has two distinct roots α and β. Find the value of $-\alpha^2 - 4\beta + 5$. [4.9%]

$x^2 - 4x + 5 = 0$ 有兩個相異的根 α 和 β。求 $-\alpha^2 - 4\beta + 5$ 的值。

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11) If $60^x = 2$, $60^y = 3$ and $60^z = 5$, find the value of 2^{6a}, where $a = \dfrac{1+x-y}{x-y-z+1}$. [5.1%]

若 $60^x = 2$、$60^y = 3$ 和 $60^z = 5$，求 2^{6a} 的值，其中 $a = \dfrac{1+x-y}{x-y-z+1}$。

12) Given that real numbers p, q and r satisfy the following system of equations, find the roots of the equation $qx^2 + rx - p = 0$. [5.2%]

已知實數 p、q 和 r 滿足下列的聯立方程，求 $qx^2 + rx - p = 0$ 的根。

$$\begin{cases} p+q = 12 \\ -r^2 + 6r = 45 - pq \end{cases}$$

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13) Given that the denominator in each term in the following equation is an integer, find the negative solution to the following equation. [6.3%]

已知下列方程中每項的分母均為整數，求下列方程的負數解。

$$\frac{1}{x^2 - 19x - 40} + \frac{2}{x^2 - 19x - 38} + \frac{1}{41 - x^2 + 19x} = 0$$

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14) Define $[a]$ to be the greatest integer that is not greater than a. Find the sum of all the negative solutions to the following equation. [6.4%]

定義 $[a]$ 為不大於 a 的最大整數。求下列方程式所有負數解之和。

$$8x - 7[x] = 4$$

15) Given that N is not a square number and it has m factors. [6.5%]

Express the sum of all $\dfrac{1}{f-\sqrt{N}}$, where f is a factor of N, in terms of m and N.

已知 N 不是正方形數及有 m 個因數。

以 m 和 N 表示所有 $\dfrac{1}{f-\sqrt{N}}$ 的總和，其中 f 為 N 的因數。

16) Let x be a real number. [6.6%]

Find the greatest value of $\sqrt{(3-x)^2+(2+x^2)^2}-\sqrt{x^2+(-4+x^2)^2}$.

設 x 為實數。求 $\sqrt{(3-x)^2+(2+x^2)^2}-\sqrt{x^2+(-4+x^2)^2}$ 的最大值。

17) Find the real value of $(7+5\sqrt{2})^{\frac{2}{3}}+(7-5\sqrt{2})^{\frac{2}{3}}$. [6.7%]

求 $(7+5\sqrt{2})^{\frac{2}{3}}+(7-5\sqrt{2})^{\frac{2}{3}}$ 的實數值。

18) A 63 units × 77 units × 99 units solid cuboid is made by gluing unit cubes (cubes of side 1 unit) together. An internal diagonal of the solid is a straight line drawn from one vertex to the furthest vertex. How many unit cubes does the diagonal pass through the interior of? [6.8%]
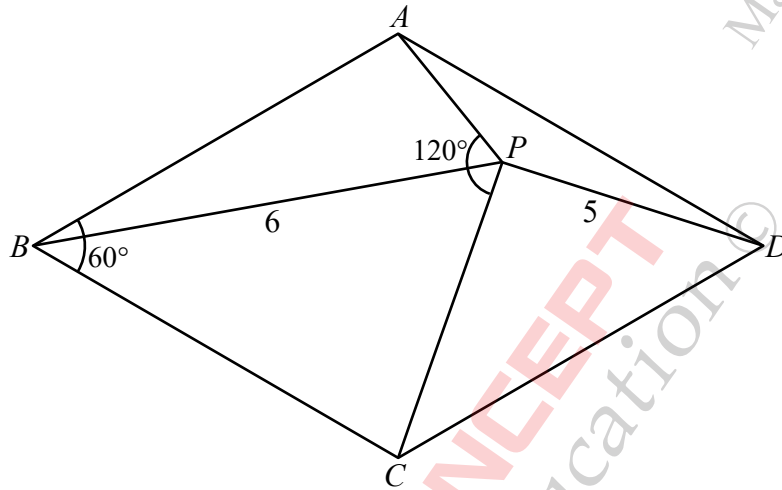
一個 63 單位 × 77 單位 × 99 單位的實心長方體是由黏合多個單位正方體（邊長為 1 單位的正方體）而成的。長方體的內部對角線是連接一頂點至與其相距最遠的頂點的直線。該對角線穿過多少個單位正方體的內部？

19) *ABCD* is a rhombus, where $\angle B = 60°$. *P* is a point inside *ABCD* such that $\angle APC = 120°$, *BP* = 6 units and *DP* = 5 units. Find the difference between the lengths of *AP* and *CP*. [6.9%]

ABCD 是個菱形，其中 $\angle B = 60°$。*P* 是 *ABCD* 內的一點，使 $\angle APC = 120°$、*BP* = 6 單位和 *DP* = 5 單位。求 *AP* 和 *CP* 的長度之差。

A

120° *P*

6 5

B 60° *D*

C

20) Find the least integer *x* that is not less than 8, for which $(x + 1)^3 - x^3$ is a perfect square. [7%]

若 $(x + 1)^3 - x^3$ 是完全平方，求不少於 8 的最小整數 *x*。

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ANSWER SHEET

○ IDCHECK   ○ ABSENT

	ANSWER			ANSWER	
1	$-\dfrac{1}{2}$ or -0.5	○	11	1600	○
2	$\dfrac{k - kx}{1 + x}$	○	12	$\dfrac{-1 \pm \sqrt{17}}{4}$ or $-\dfrac{1}{4} \pm \dfrac{\sqrt{17}}{4}$	○
3	R	○	13	-2	○
4	$-\log_3 29$	○	14	$-\dfrac{15}{4}$ or -3.75	○
5	$\dfrac{1}{2} + \dfrac{3}{2}i$ or $\dfrac{1+3i}{2}$ or $0.5 + 1.5i$	○	15	$-\dfrac{m\sqrt{N}}{2N}$	○
6	D	○	16	$3\sqrt{5}$ or $\sqrt{45}$	○
7	517	○	17	6	○
8	$\dfrac{13}{75}$	○	18	213	○
9	-45	○	19	$\dfrac{8\sqrt{3}}{3}$ or $\dfrac{\sqrt{192}}{3}$	○
10	-6	○	20	104	○

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