MathConception 2024

S3

Question Booklet

問題簿

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. **DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
2. **If the information printed on your answer sheet is not correct, please inform the invigilator immediately.**
3. **Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.**
4. **Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.**
5. **Rough-work sheets provided will be collected at the end of the contest but they will not be marked.**
6. **Diagrams in this question booklet are not necessarily drawn to scale.**

限時：1 小時
不允許使用計算機。

比賽須知：

1. 未宣布開始前，切勿翻閱此問題簿。
2. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。
3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。
4. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。
5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。
6. 此問題簿的附圖不一定依比例繪成。

1) If $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$, find $\frac{x}{y}$. [3%]

若 $\begin{cases} x+y=6 \\ x-y=2 \end{cases}$，求 $\frac{x}{y}$。

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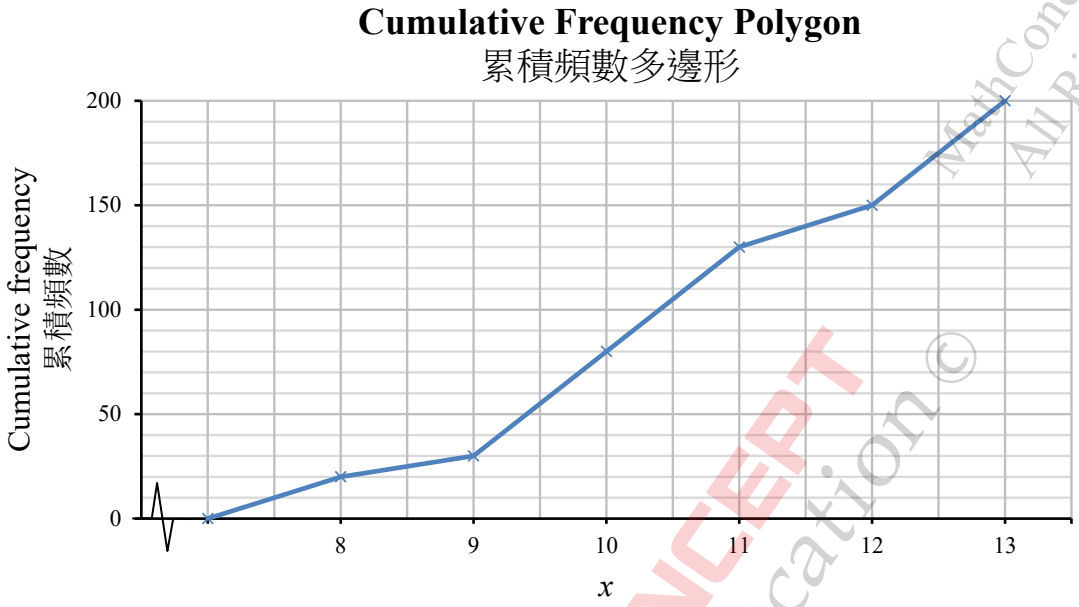
2) Find the class width of the following frequency distribution table. [3.1%]

求下列頻數分佈表的組距。

Class interval 組區間	141 – 151	152 – 162	163 – 173	174 – 184	185 – 195
Class mark 組中點	146	157	168	179	190
Frequency 頻數	4	14	10	7	2

3) Find the number of modal class(es) of the grouped data presented in the following cumulative frequency polygon. [3.2%]

求下列累積頻數多邊形中所示的分組數據的眾數組數目。



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4) Factorize $(x + 2)^2 + 2xy + 6x + 4y + 12 + (y + 3)^2$. [3.3%]

因式分解 $(x + 2)^2 + 2xy + 6x + 4y + 12 + (y + 3)^2$。

5) A sum of $12 500 is deposited in a bank at a simple interest rate of 4% p.a.. After a certain period of time, an amount of $13 500 is received from the bank. How many years elapsed during the deposit period? [3.4%]

把 $12 500 存入銀行，年利率為 4%，並以單利息計算。存款一段時期後，得到本利和 $13 500。該存款期是多少年？

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6) Evaluate $\dfrac{0.5^{-n}}{0.5^{1-n}+0.5^{-n-1}}$, where n is an integer. [3.5%]

計算 $\dfrac{0.5^{-n}}{0.5^{1-n}+0.5^{-n-1}}$，其中 n 為一個整數。

7) On a rectangular coordinate plane, find the area of the convex polygon with vertices (9, 2), (5, 10), (–5, 2), (8, –4). [3.6%]

在直角坐標平面上，求頂點為 (9, 2)、(5, 10)、(–5, 2)、(8, –4) 的凸多邊形面積。

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8) If the ratio of the total surface area of a sphere to that of a hemisphere is 3 : 1, find the ratio of the volume of the sphere to that of the hemisphere. [3.7%]

若某球體與某半球體的總表面面積之比為 3 : 1，求該球體與該半球體的體積之比。

9) The sides of a triangle are 20, $2x$ and $20 - x$ respectively, where x is an integer. Find the sum of all possible value(s) of x. [4.8%]

三角形的邊長分別為 20、$2x$ 和 $20 - x$，其中 x 為一個整數。求 x 的所有可能值之和。

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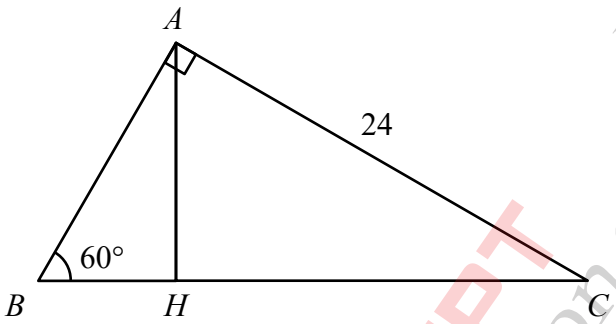
10) Andy and Betty are playing a game of tossing a fair coin. If the coin shows a head, Andy gets a point; if the coin shows a tail, Betty gets a point. The first person to get 10 points wins. Given that Betty is leading with 8 points to Andy's 7 points now, find the probability of Andy winning the game. [4.9%]

小明、小芬正在進行一場遊戲，擲一枚勻稱硬幣。若硬幣正面朝上，小明得一分；若硬幣反面朝上，小芬得一分。最先獲得 10 分的人獲勝。已知小芬現在以 8 比 7 領先小明，求小明在該遊戲中獲勝的概率。

11) The figure shows a right-angled triangle ABC, where $AC = 24$. AH is an altitude of $\triangle ABC$. Find the length of BH. [5.1%]

(Leave your answer in the simplest surd form.)

圖中所示為一個直角三角形 ABC，其中 $AC = 24$。AH 為$\triangle ABC$ 的高線。求 BH 的長度。（以最簡根式表示答案。）



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12) If $x^2 + y^2 - 6xy = 0$, find the value of $\dfrac{x+y}{x-y}$ for any positive x, y and $x > y$. [5.2%]

對於任意正數 x、y 且 $x > y$，若 $x^2 + y^2 - 6xy = 0$，求 $\dfrac{x+y}{x-y}$ 的值。

13) If $x^{12}+x^8+x^4+1=0$, find the value of the following expression. [6.3%]

若$x^{12}+x^8+x^4+1=0$，求下列算式的值。

$$x^{100}+x^{80}+x^{60}+x^{40}+x^{20}+x^{12}+x^8-24$$

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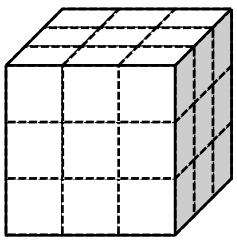
14) The total surface area of a cube is 54 square units. It can be cut into 27 identical small cubes. Find the difference between the largest and smallest possible total surface areas of the remaining part after removing 10 small cubes. [6.4%]

一個總表面面積為 54 平方單位的正方體，可被分割為 27 個相等的小正方體。求移去 10 個小正方體後，餘下部分的最大和最小的可能總表面面積之差。



15) If $45+\cfrac{1}{45+\cfrac{1}{90+\cfrac{1}{45+\cfrac{1}{90+\cfrac{1}{45+\cfrac{1}{90+\cdots}}}}}}=\sqrt{x}$, find x. [6.5%]

若 $45+\cfrac{1}{45+\cfrac{1}{90+\cfrac{1}{45+\cfrac{1}{90+\cfrac{1}{45+\cfrac{1}{90+\cdots}}}}}}=\sqrt{x}$，求 x。

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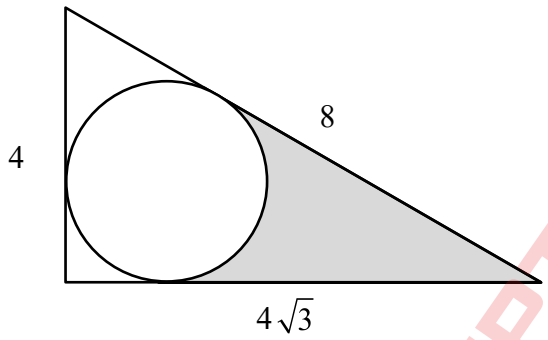
16) Find the sum of all common factors of 39 806 208 and 898 560. [6.6%]

求 39 806 208 和 898 560 的所有公因數之和。

17) The figure shows a circle inscribed in a triangle with sides of 4, 8 and $4\sqrt{3}$, find the area of the shaded region. [6.7%]
(Take π as 3 and leave your answer in the simplest surd form.)

如圖所示，一個圓內接於一個邊長為 4、8 和 $4\sqrt{3}$ 的三角形，求陰影部分的面積。（取 π 為 3，並以最簡根式表示答案。）



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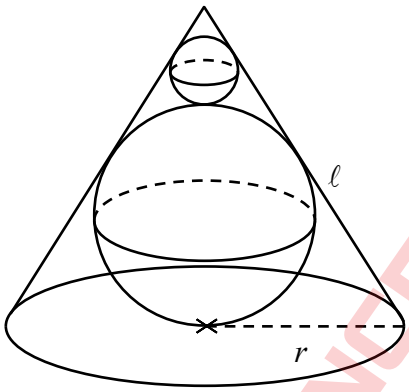
18) When $x = a$ and $y = b$, the following expression attains its minimum value. Find the product of all possible values of a and b. [6.8%]

當 $x = a$ 及 $y = b$ 時下列算式達至其最小值，求 a 和 b 的所有可能值之積。

$$4x^4 - 12x^3y - 20x^3 + 18x^2y^2 + 159x^2 - 12xy^3 - 10xy - 540x + 3y^4 + 5y^2 + 850$$

19) In the figure, the larger sphere is inscribed in the right circular cone with radius r and slant height ℓ. The slant height of the cone is tangent to the smaller sphere, and the spheres touch each other. Find the ratio of the surface area of the smaller sphere to that of the larger sphere in terms of r and ℓ. [6.9%]

在圖中，大球體內接於半徑為 r、斜高為 ℓ 的直立圓錐。該圓錐的斜高與小球體相切且兩球體相互接觸。以 r 和 ℓ 表示該小球體與該大球體的表面面積之比。



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20) How many different arrangements of the letters in the word 'attentive' are there if any two identical letters cannot be placed together? [7%]

若任何兩個相同的字母都不能並排放置，則「attentive」一詞中的字母有多少種不同的排列方式？

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MATH 數學·思維大激鬥 CONCEPTITION

REG NO			S3
NAME			
GRADE			
SEAT			

ANSWER SHEET

○ IDCHECK ○ ABSENT

	ANSWER			ANSWER	
1	2		11	$4\sqrt{3}$	
2	11		12	$\sqrt{2}$	
3	3		13	-25	
4	$(x+y+5)^2$		14	36	
5	2		15	2027	
6	$\frac{2}{5}$ or 0.4		16	$286\ 160$	
7	98		17	$10\sqrt{3}-12$	
8	$27:4$		18	25	
9	91		19	$\left(\frac{\ell-r}{\ell+r}\right)^2$	
10	$\frac{5}{16}$ or 0.3125		20	10200	

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