



# MathConceptition

2024

# S1

# Question Booklet

# 問題簿

**Time: 1 hour**

**Calculators are NOT permitted.**

**Instructions:**

- 1. DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
- 2. If the information printed on your answer sheet is not correct, please inform the invigilator immediately.**
- 3. Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.**
- 4. Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.**
- 5. Rough-work sheets provided will be collected at the end of the contest but they will not be marked.**
- 6. Diagrams in this question booklet are not necessarily drawn to scale.**

**限時：1 小時**

**不允許使用計算機。**

**比賽須知：**

- 1. 未宣布開始前，切勿翻閱此問題簿。**
- 2. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。**
- 3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。**
- 4. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。**
- 5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。**
- 6. 此問題簿的附圖不一定依比例繪成。**

- 1) The difference in weights between David and Ederson was 7.3 kg. After one month, Ederson gains 4.2 kg and David gains 1.7 kg. If David is always lighter than Ederson, what is the difference in weights between David and Ederson now in kg? [3%]

小明和大明的體重相差 7.3 公斤。一個月後，大明的體重增加了 4.2 公斤，小明則增加了 1.7 公斤。若小明一向比大明輕，小明和大明的體重現在相差多少公斤？

- 
- 2) The general term of a sequence is  $T_n = \frac{3}{2n+5} - n$ . Find the difference between the 2<sup>nd</sup> and 5<sup>th</sup> term in the sequence. [3.1%]

一個數列的通項是  $T_n = \frac{3}{2n+5} - n$ 。求數列的第二項和第五項之差。

- 3) Raymond bought a watch at \$48 000 five years ago. If he now sells the watch at \$32 640, what is the loss per cent? [3.2%]

家俊五年前以\$48 000 購買了一隻手錶。若他以\$32 640 的價錢轉售該手錶，求虧蝕百分率。

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- 4) In the algebraic expression  $\frac{4a^2b}{c}$ , where  $a, b, c > 0$ , the value of  $a$  increases [3.3%]

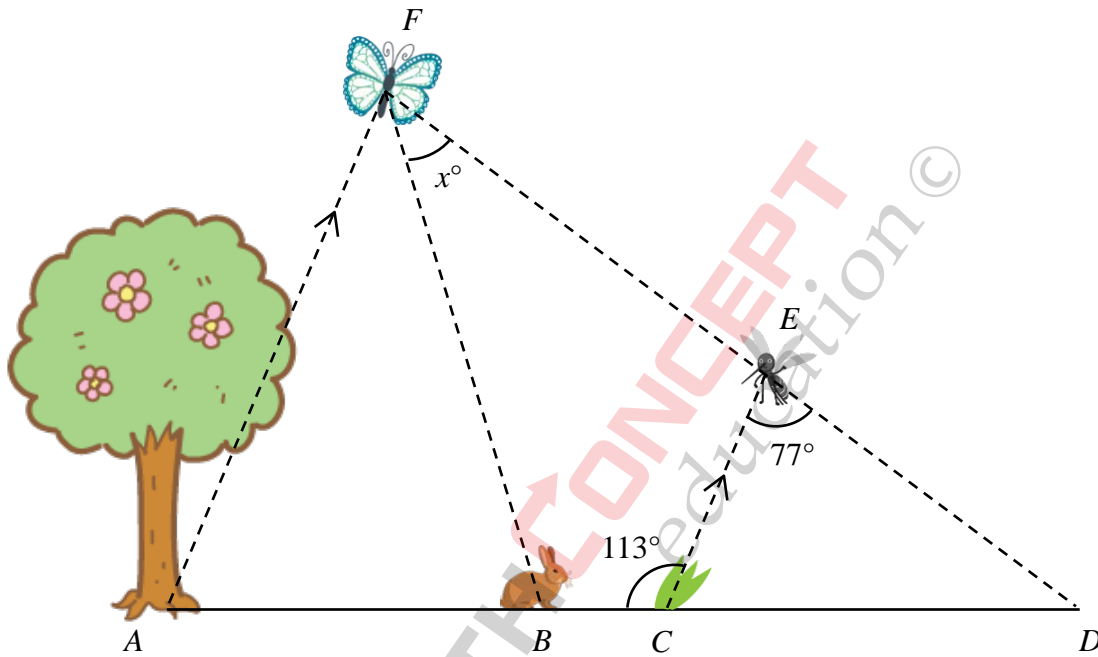
by 25%, and the value of  $b$  decreases by 60%, while the value of  $c$  remains unchanged. What is the percentage change of the value of the algebraic expression?

在代數式  $\frac{4a^2b}{c}$  中，其中  $a, b, c > 0$ ， $a$  的值增加 25%、 $b$  的值減少 60%、 $c$  的值保持不變。該代數式的值的百分變化是多少？

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- 5) The diagram below shows the scenery in the garden.  $ABCD$  is the horizontal ground and  $FED$  is a straight line, where  $AF \parallel CE$ . Given  $\angle FBA$  is an acute angle and  $\angle FBA = 2\angle FDA$ , find  $x$ . [3.4%]

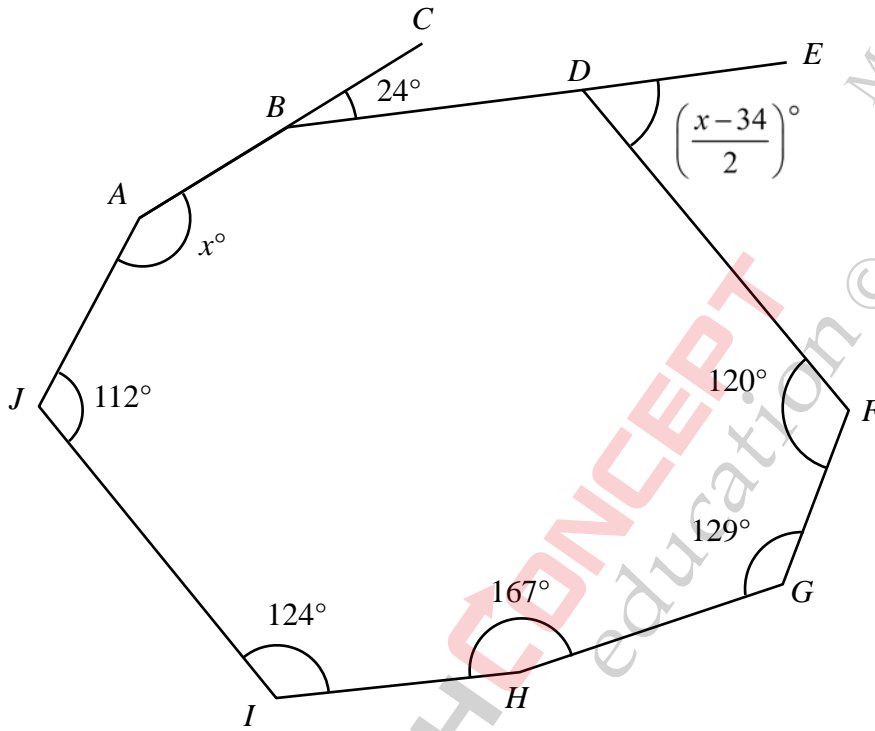
下圖所示為一個花園的景色。 $ABCD$ 是在同一水平線上， $FED$ 是一條直線，其中 $AF \parallel CE$ 。若 $\angle FBA$ 是一個銳角，且 $\angle FBA = 2\angle FDA$ ，求 $x$ 。



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- 6) In the figure,  $ABDFGHIJ$  is an octagon.  $ABC$  and  $BDE$  are straight lines. If the sum of interior angles of the octagon is  $1080^\circ$ , find  $x$ . [3.5%]

在圖中， $ABDFGHIJ$  是一個八邊形。  $ABC$  和  $BDE$  都是直線。 若該八邊形的內角總和是  $1080^\circ$ ，求  $x$ 。



- 7) The following back-to-back stem-and-leaf diagram shows the scores of two groups of people in a competition. Given that the sum and the product of the lowest and highest scores in Group A are multiples of 3, and the difference between the lowest and highest scores in Group A is less than 45 marks. If none of the members in each group has the same score, find the difference between the lowest scores of Group A and B. [3.6%]

以下是一個背靠背幹葉圖，顯示了兩組人在一場遊戲中得到的分數。已知 A 組最高和最低得分之和及積都是 3 的倍數，且 A 組最高和最低得分之差少於 45 分。若同組組員的分數都沒有重複，求 A 組和 B 組最低得分之差。

Scores in a competition  
一場遊戲所得的分數

Group A A 組	Stem (10 marks) 幹 (10 分)	Group B B 組
<u>Leaf (1 mark)</u> 葉 (1 分)		<u>Leaf (1 mark)</u> 葉 (1 分)
	0	6
8 x	1	3 4 9
9 6 3	2	2 5
8 0	3	4
2	4	1 5
7 2	5	
	6	6

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- 8)  $x$  and  $y$  are positive integers, where  $\angle A = x^\circ$  and  $\angle B = y^\circ$  are supplementary angles. Find the sum of all possible values of  $x$  and all possible values of  $y$ . [3.7%]

$x$  和  $y$  都是正整數，且  $\angle A = x^\circ$  和  $\angle B = y^\circ$  互為補角，求  $x$  的所有可能值和  $y$  的所有可能值之和。

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- 9) A bottle of 200 mL of red wine contains 14.5% alcohol. The teacher puts the red wine into an apparatus in the laboratory such that the water content in the wine lessen while the amount of alcohol remains unchanged. If the red wine contains 25% of alcohol after the experiment, what is the new volume of red wine in litres? [4.8%]

一瓶 200 毫升的紅酒內有酒精 14.5%。老師將這一瓶紅酒放進實驗室的儀器中，使紅酒內的水份變少，而酒精的量不變。若紅酒在實驗結束後的酒精含量是 25%，紅酒的新體積是多少升？

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- 10) Find the constant term in the expression [4.9%]  
 $\left(x - \frac{1}{n}\right) \dots \left(x - \frac{1}{4}\right) \left(x - \frac{1}{3}\right) \left(x - \frac{1}{2}\right) (x-1)(x-2)(x-3)(x-4) \dots (x-n),$

where  $n$  is a positive integer.

求以下算式的常數項

$$\left(x - \frac{1}{n}\right) \dots \left(x - \frac{1}{4}\right) \left(x - \frac{1}{3}\right) \left(x - \frac{1}{2}\right) (x-1)(x-2)(x-3)(x-4) \dots (x-n),$$

其中  $n$  是正整數。

- 11) Define  $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ . For example,  $|3| = 3$  and  $|-4| = 4$ . [5.1%]

Find the value of  $|2a + 10| + |2a - 4| + 6$ , where  $-3 \leq a \leq 2$ .

定義  $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ 。例如， $|3| = 3$  及  $|-4| = 4$ 。

求  $|2a + 10| + |2a - 4| + 6$  的值，其中  $-3 \leq a \leq 2$ 。



- 12) Given that  $\frac{x}{6} = \frac{y}{15} = \frac{z}{5}$  and  $\frac{5x+2y}{10} + \frac{7z+6}{2} = 50$ , find the value of  $x + y + z$ . [5.2%]

已知  $\frac{x}{6} = \frac{y}{15} = \frac{z}{5}$  和  $\frac{5x+2y}{10} + \frac{7z+6}{2} = 50$ ，求  $x + y + z$  的值。

- 13) The ratio of the width to the length of a rectangle is 4 : 5. If the width decreases by 12 cm, then the ratio becomes 2 : 5. What is the area of the new rectangle in  $\text{cm}^2$ ? [6.3%]

一個長方形的闊和長的比例是 4 : 5。若闊減少 12 cm，比例則變成 2 : 5。新長方形的面積是多少  $\text{cm}^2$ ？

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- 14) Find the units digit of the value of the following expression. [6.4%]

求以下數式的值的個位數。

$$22^3 + 23^3 + 24^3 + 25^3 + \cdots + 1002^3 + 1003^3 + 1004^3$$

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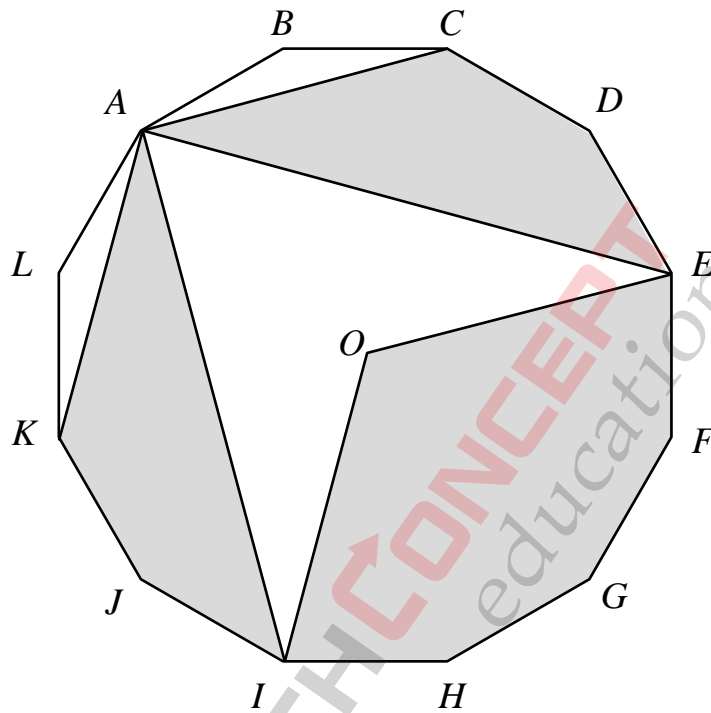
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- 15) It is known that the sum of 42 positive integers  $a_1, a_2, \dots, a_{42}$  arranged in ascending order is 2024. If all the 42 positive integers are distinct, what is the largest value of  $a_{40}$ ? [6.5%]

已知由小到大排列的 42 個正整數  $a_1, a_2, \dots, a_{42}$  之和是 2024。  
若該 42 個正整數都是不同的， $a_{40}$  的最大值是多少？

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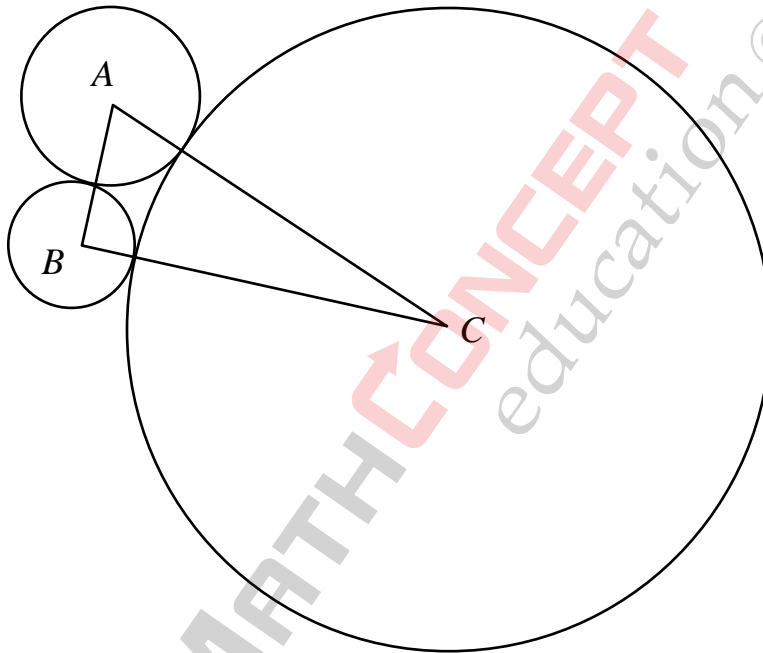
- 16) The figure below is a regular 12-sided polygon  $ABCDEFGHIJKL$ , where  $O$  is the centre. What percentage of area of the white parts is the area of the shaded parts? [6.6%]

以下是一個正十二邊形  $ABCDEFGHIJKL$ ，其中  $O$  是中心。陰影部分的面積佔白色部分的百分之幾？



- 17) The following figure is formed by three circles of different sizes. Points  $A$ ,  $B$ ,  $C$  are the centres of the circles respectively. If  $AB = 10$  units,  $BC = 24$  units and  $AC = 26$  units, find the sum of areas of the three circles in square units. (Express your answer in terms of  $\pi$ .) [6.7%]

以下的圖形是由三個不同大小的圓組合而成。A 點、B 點和 C 點分別為該三個圓的圓心。若  $AB = 10$  單位， $BC = 24$  單位及  $AC = 26$  單位，三個圓的面積總和是多少平方單位？（答案以  $\pi$  表示。）



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- 18) There is a 6-digit number  $\overline{abcabc}$ , where  $a$ ,  $b$ , and  $c$  are distinct 1-digit primes, such that  $\overline{abcabc}$  is the largest. Find the sum of all factors of  $\overline{abcabc}$ . [6.8%]

已知某六位數  $\overline{abcabc}$ ，且  $a$ ， $b$  和  $c$  都是不同的一位質數，使得  $\overline{abcabc}$  是最大的。求  $\overline{abcabc}$  的所有因數之和。

- 19) Define  $[x]$  as the result of rounding down  $x$  to the nearest integer, where  $x$  is a real number. [6.9%]

If  $\left[ A + \frac{4}{100} \right] + \left[ A + \frac{6}{100} \right] + \left[ A + \frac{8}{100} \right] + \dots + \left[ A + \frac{88}{100} \right] + \left[ A + \frac{90}{100} \right] = 288$ , for some real number  $A$ , find the value of  $\left[ 5A + \frac{98}{100} \right]$ .

定義  $[x]$  是將  $x$  下捨入至最接近的整數的結果，其中  $x$  是一個實數。

若  $\left[ A + \frac{4}{100} \right] + \left[ A + \frac{6}{100} \right] + \left[ A + \frac{8}{100} \right] + \dots + \left[ A + \frac{88}{100} \right] + \left[ A + \frac{90}{100} \right] = 288$ ，且  $A$  是某實數，求  $\left[ 5A + \frac{98}{100} \right]$  的值。

- 20) Given that  $a, b, c$  and  $d$  are non-zero real numbers, where  $\frac{a+b+c-d}{d} = \frac{a+b-c+d}{c} = \frac{a-b+c+d}{b} = \frac{-a+b+c+d}{a}$ . Find the sum of all possible values of  $\frac{(a+b+c)(a+b+d)(a+c+d)(b+c+d)}{abcd}$ . [7%]

已知  $a, b, c$  和  $d$  都是非零實數，其中

$$\frac{a+b+c-d}{d} = \frac{a+b-c+d}{c} = \frac{a-b+c+d}{b} = \frac{-a+b+c+d}{a}。$$

求  $\frac{(a+b+c)(a+b+d)(a+c+d)(b+c+d)}{abcd}$  的所有可能值之和。

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REG NO			<b>S1</b>
NAME			
GRADE			
SEAT			

**ANSWER SHEET**

IDCHECK

ABSENT

ANSWER		ANSWER	
1	9.8	11	20
2	$3\frac{2}{15} / \frac{47}{15}$	12	52
3	32(%)	13	360
4	-37.5%/-37.5(%) Decreased by 37.5%	14	9
5	36°	15	413
6	150 (no unit)	16	200 (%)
7	9 (marks)	17	452π (sq units)
8	32 220	18	1 354 752
9	0.116 (L)	19	33
10	-1	20	82

