MathConceptition
2023

S1

Question Booklet

問題簿

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. **DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
2. **If the information printed on your answer sheet is not correct, please inform the invigilator immediately.**
3. **Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.**
4. **Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.**
5. **Rough-work sheets provided will be collected at the end of the contest but they will not be marked.**
6. **Diagrams in this question booklet are not necessarily drawn to scale.**

限時：**1 小時**
不允許使用計算機。

比賽須知：

1. 未宣布開始前，切勿翻閱此問題簿。

2. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。

3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。

4. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。

5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。

6. 此問題簿的附圖不一定依比例繪成。

1) Round down $54^2 - 6^2$ to 2 significant figures. [3%]

把 $54^2 - 6^2$ 下捨入至二位有效數字。

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2) The marked price of a table is x and it is sold at a discount of 25%, which is equivalent to $260. Find the value of x. [3.1%]

一張桌子的標價是 x 元。桌子最後以 25% 的折扣百分比售出，即折扣了 260 元。求 x 的值。

3) Given a is $\frac{5}{6}$ of b. If a satisfies the equation $1.5a + \frac{a}{17} = 530\%$, find the value   [3.2%]
of $2b$.

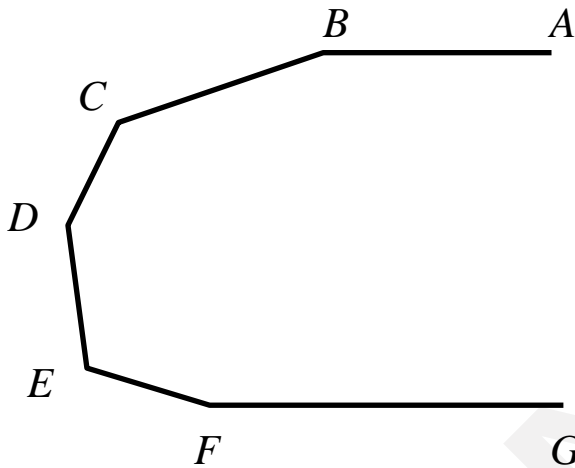已知 a 是 b 的 $\frac{5}{6}$。若 a 滿足方程 $1.5a + \frac{a}{17} = 530\%$，求 $2b$ 的值。

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4) Consider the sequence of triangular numbers 1, 3, 6, … . Find the sum of the   [3.3%]
first 8 terms.

考慮三角形數數列 1, 3, 6, … 。求首 8 項之和。

5) In the figure, $AB \parallel FG$. [3.4%]

If $\angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFG = a°$, find the value of a.

圖中，$AB \parallel FG$。若 $\angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFG = a°$，
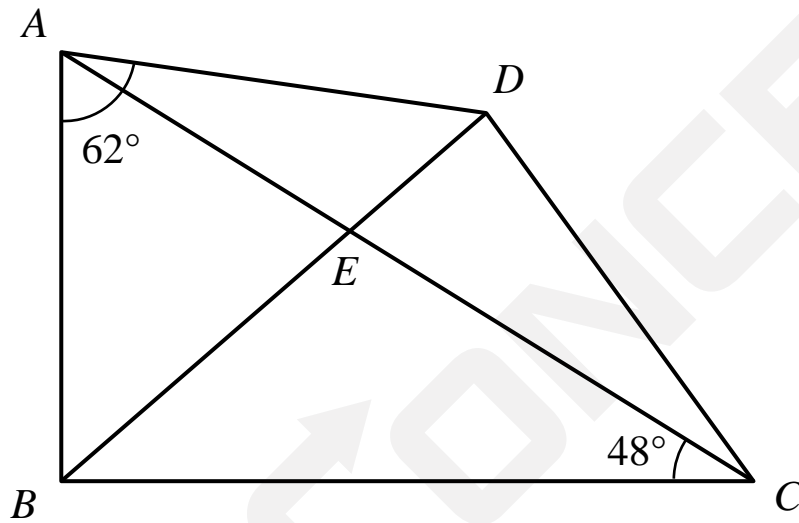求 a 的值。

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6) Simplify $\dfrac{1829}{10013}$ to its simplest form. [3.5%]

化簡 $\dfrac{1829}{10013}$ 至最簡分數。

7) In the figure, *ABCD* is a quadrilateral. *AC* and *BD* intersect at *E* and [3.6%]
AB = AD. Given $\angle BAD = 62°$, $\angle BCE = 48°$, and $\angle ABC$ is a right angle.
If $\angle AED = x°$, find the value of *x*.

圖中，*ABCD* 是一個四邊形。*AC* 和 *BD* 相交於 *E*，且 *AB = AD*。
已知 $\angle BAD = 62°$，$\angle BCE = 48°$，$\angle ABC$ 是一個直角。
若 $\angle AED = x°$，求 *x* 的值。

8) The following is a pentagon on a rectangular coordinate plane. If the area of the pentagon is 183.25 square units, find the value of x. [3.7%]

以下是一個在直角坐標平面上的五邊形。若該五邊形的面積是 183.25 平方單位，求 x 的值。

$A(-11, 4)$ $E(9.5, 4)$

$B(-7, -5)$ $D(6, -5)$

$C(0, -x)$
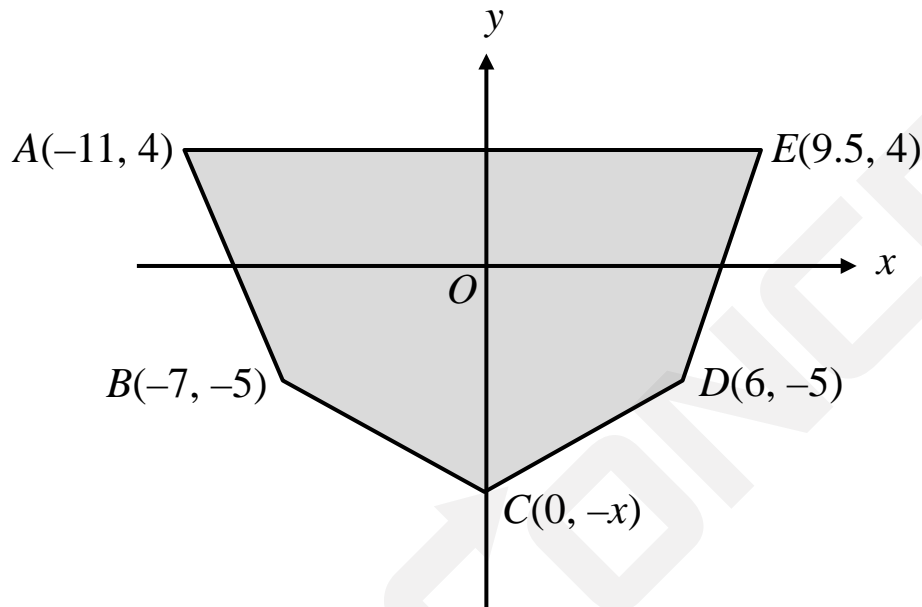
9) In $\triangle ABC$, $AB = AC$ and $\angle ABC = 69°$. If there are 13 lines drawn from point A to line BC such that $\angle CAA_1 = \angle A_1AA_2 = \angle A_2AA_3 = \ldots = \angle A_{13}AB$, how many times of the size of $\angle CAA_1$ is the size of $\angle ACB$? [4.8%]

在$\triangle ABC$，$AB = AC$，且$\angle ABC = 69°$。若有 13 條線從 A 點延伸至線段 BC，使得$\angle CAA_1 = \angle A_1AA_2 = \angle A_2AA_3 = \ldots = \angle A_{13}AB$，$\angle ACB$ 的大小等於$\angle CAA_1$的大小的多少倍？
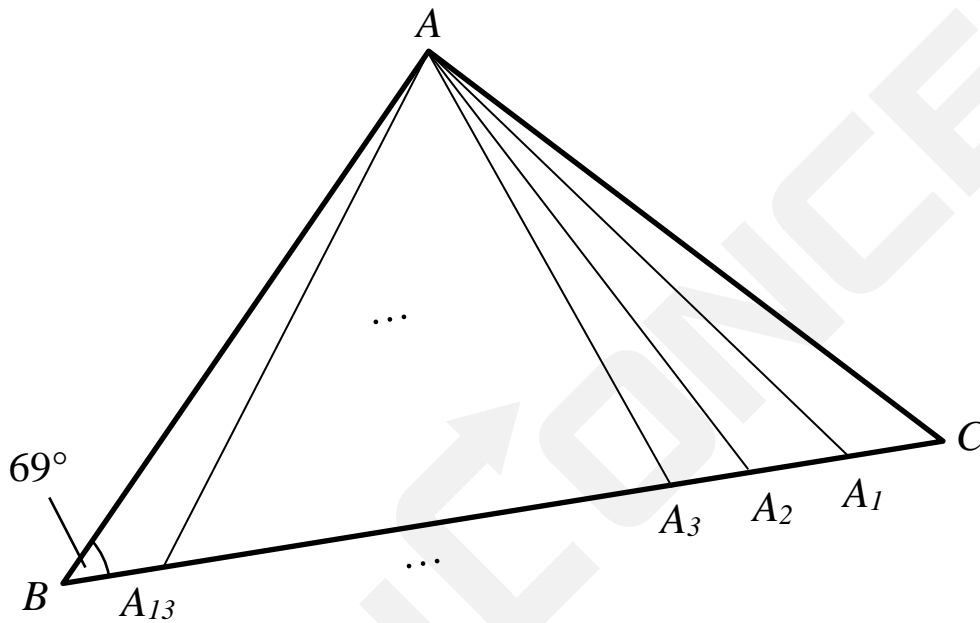
10) Find the smallest 3-digit number which has exactly 5 distinct factors. [4.9%]

求剛好擁有 5 個不同因數的最小三位數。

11) 111132 is divisible by a positive integer a, and its quotient is a cubic number. Find the smallest value of a. [5.1%]

111132 可以被正整數 a 整除，其商是一個立方數。求 a 的最小值。

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12) Given $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$. For example, $|3| = 3$ and $|-4| = 4$. It is known [5.2%]

that a, b, c, d and e are real numbers, where $\dfrac{a}{|a|} < 0$, $b = 0$, the difference

between b and c is 3 units and d is a reciprocal of e. Find the value of

$2b^2 - \left(\dfrac{a}{|a|}\right)\left[-1 + \left(\dfrac{a}{|a|}\right)\right] - (c - b)^2 + 6de$.

已知 $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$，例如，$|3| = 3$ 及 $|-4| = 4$。得知 a、b、c、d 和 e

是實數，且 $\dfrac{a}{|a|} < 0$、$b = 0$、b 與 c 之差是 3 單位、d 和 e 互為倒數。

求 $2b^2 - \left(\dfrac{a}{|a|}\right)\left[-1 + \left(\dfrac{a}{|a|}\right)\right] - (c - b)^2 + 6de$ 的值。

13) What is the 2023$^{\text{rd}}$ digit after the decimal point of $3.\dot{1}42\dot{5}+1.\dot{3}\dot{4}$? [6.3%]

$3.\dot{1}42\dot{5}+1.\dot{3}\dot{4}$的小數點後的第 2023 個位的數字是多少？

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14) What is the sum of digits of $2^2 \times 3^2 \times 5^8 \times 8^2 \times 15^3 \times 20^8$? [6.4%]

$2^2 \times 3^2 \times 5^8 \times 8^2 \times 15^3 \times 20^8$所有數字之和是多少？

15) When a positive integer a is divided by 144, the remainder is 22. When another positive integer b is divided by 48, its remainder is 13. Find the remainder of $(4ab)^3 - 99$ dividing by 48. [6.5%]

某正整數 a 除以 144 的餘數是 22，另一正整數 b 除以 48 的餘數是 13。求$(4ab)^3 - 99$ 除以 48 的餘數。

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16) Given $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$. If $x < 0$, simplify $\dfrac{\left|2|x|+5x\right|}{|3x-8|-|3x|}$ and express your answer in terms of x. [6.6%]

已知$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$。若 $x < 0$，化簡 $\dfrac{\left|2|x|+5x\right|}{|3x-8|-|3x|}$，並把答案以 x 表示。

17) Let $A = \dfrac{1}{\dfrac{1}{2024} + \dfrac{1}{2023} + \cdots + \dfrac{1}{2001} + \dfrac{1}{2000}}$. When the integral part of A is [6.7%]

divided by 48, what is the remainder?

設 $A = \dfrac{1}{\dfrac{1}{2024} + \dfrac{1}{2023} + \cdots + \dfrac{1}{2001} + \dfrac{1}{2000}}$。$A$ 的整數部分除以 48 後，餘

數是多少？

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18) There are two distinct natural numbers x and y. The sum of x and y is a [6.8%]
square number, where adding 199 to the sum is another square number. If y is
2 times of x, find the value of x.

有兩個不同的自然數 x 和 y。x 和 y 之和是一個正方形數，而它們的和
再加上 199 是另一個正方形數。若 y 是 x 的 2 倍，求 x 的值。
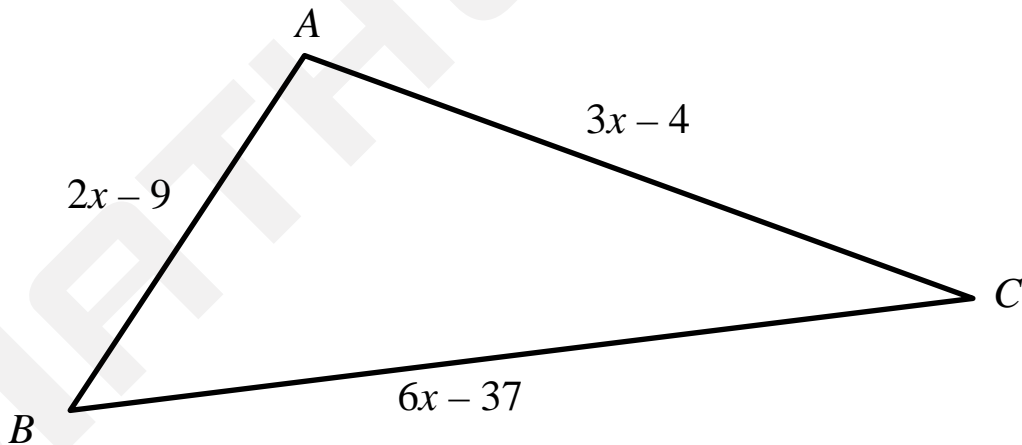
19)   Find the sum of factors of 22200.   [6.9%]

求 22200 的所有因數之和。

20)   In the scalene triangle *ABC* below, $AB = 2x - 9$, $AC = 3x - 4$ and   [7%]
$BC = 6x - 37$, where *x* is a positive integer. Find the sum of all possible
values of *x*.

在以下的不等邊三角形 *ABC* 中，$AB = 2x - 9$，$AC = 3x - 4$，且
$BC = 6x - 37$，其中 *x* 是一個正整數。求 *x* 的所有可能值之和。

End of paper
全卷完

○ IDCHECK   ○ ABSENT

	ANSWER			ANSWER	
1	2800	○	11	12	○
2	1040	○	12	–5	○
3	$8.16 / 8\frac{4}{25}$	○	13	5	○
4	120	○	14	27	○
5	720	○	15	13	○
6	$\frac{59}{323}$	○	16	$-\frac{3}{8}x$	○
7	101	○	17	32	○
8	10	○	18	3267	○
9	23	○	19	70680	○
10	625	○	20	229	○

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