MathConception 2021

S1

Question Booklet 問題簿

Name:
姓名：

Reg. No.:
登記編號：

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. **DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
2. **Write your name and registration number on the cover of this question booklet.**
3. **If the information printed on your answer sheet is not correct, please inform the invigilator immediately.**
4. **Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.**
5. **Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.**
6. **Rough-work sheets provided will be collected at the end of the contest but they will not be marked.**
7. **Diagrams in this question booklet are not necessarily drawn to scale.**

限時：1 小時
不允許使用計算機。

比賽須知：

1. 未宣布開始前，切勿翻閱此問題簿。
2. 請在此問題簿封面的適當位置寫上你的姓名及登記編號。
3. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。
4. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。
5. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。
6. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。
7. 此問題簿的附圖不一定依比例繪成。

1) Find the third term of the sequence $a_n = -n^2(n-5)^2(n+1)$. [3%]

一數列的通項為 $a_n = -n^2(n-5)^2(n+1)$，求該數列中的第 3 項。

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2) Given that $6k + \frac{2}{5} = \frac{1}{4}$, what is the value of k? [3.1%]

已知 $6k + \frac{2}{5} = \frac{1}{4}$，求 k 的值。

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3) If $3y$ is the smallest number among three consecutive odd numbers, what is the sum of the three numbers? [3.2%]

有三個連續單數，其中最小的為 $3y$。問這三個連續單數之和是多少？

4) A square with side length 12 cm has the same perimeter as an isosceles triangle. If one side of the triangle is 10 cm, how long is the longest side of the triangle? [3.3%]

一個邊長為 12 厘米的正方形，其周界與一個等腰三角形的周界相同。如果三角形的其中一條邊為 10 厘米，問三角形最長的一條邊是多少厘米？

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5) In a test of 30 questions, 4 marks were awarded for a correct answer while 2 marks were deducted for a wrong answer. No mark was deducted for not answering. If John answered 26 questions and got 18 correct, what was his total mark? [3.4%]

數學測驗有 30 題，答對一題得 4 分，答錯一題扣 2 分，沒答不扣分。如果小明答了 26 題，有 18 題答對，問他得到多少分？

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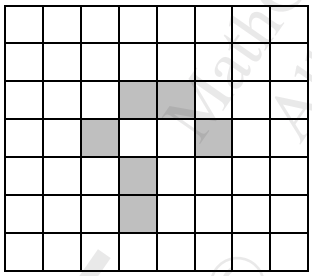
6) If the area of one side of a cube is 24 cm^2, then the volume of the cube is __?__ cm^3. [3.5%]

某正方體的其中一面面積為 24 cm^2，它的體積是 __?__ cm^3。

7) What is the minimum number of boxes should be shaded so that the figure below has rotational symmetry? [3.6%]

下圖中，最少要塗多少個格仔才能使著色部份有旋轉對稱軸？



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8) If P is 25% less than Q, then Q is ___?___% more than P. [3.7%]

如果 P 比 Q 少 25%，那麼 Q 比 P 多 ___?___%。

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9) The obtuse angle formed by the hour hand and the minute hand of a clock at 9:20 is $n°$. Find the value of n. [4.8%]

當鐘面顯示的時間是 9:20，時針與分針組成的鈍角是 $n°$。求 n 的值。

10) A car traveled from Town A to Town B, which were 240 km apart, in 3 hours. If the car's speed is increased by 25%, how many minutes will be saved to travel on the same path? [4.9%]

一輛汽車從甲城開往 240 公里外的乙城需時 3 小時。如果同一輛汽車把速度提高 25%，那麼它行駛同一路段可以節省多少分鐘？

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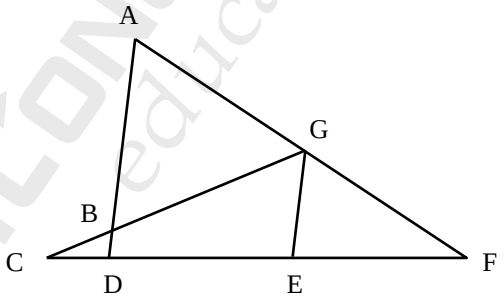
11) In the figure, AD // GE and EF = DE = 3CD. How many times of BD is AB? [5.1%]

下圖中，AD // GE 及 EF = DE = 3CD。問 AB 是 BD 的多少倍？



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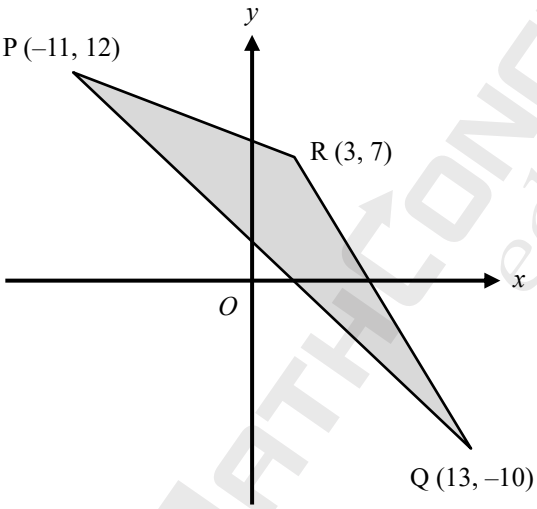
12) Find the area of the shaded region. [5.2%]

求圖中著色部份的面積。



13) Given that Figure A has 16-fold rotational symmetry and Figure B has 20-fold rotational symmetry. If Figure C is formed by overlapping Figure A and Figure B together with the same centre, what is the order of rotational symmetry of Figure C? [6.3%]

已知圖 A 是一個 16 重旋轉對稱圖形，圖 B 是一個 20 重旋轉對稱圖形。若把圖 A 和圖 B 以中心點重疊來組成圖 C，問圖 C 是多少重旋轉對稱圖形？

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14) There are three different colors of marbles in a bottle, 26% of the marbles are blue. If the number of red marbles is 48% of the green marbles, then ___?___% of the marbles in the bottle are green. [6.4%]

瓶內有紅、藍、綠三種顏色的波子，其中藍色波子佔了 26%。如果紅色波子的數量是綠色波子的 48%，即綠色波子佔全部波子的 ___?___%。

15) What is the remainder when the 2020-digit number 20212021…2021 is divided by 13? [6.5%]

當 2020 位數 20212021…2021 除以 13 時，餘數是多少？

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16) Find the value of $\frac{43^2+86^2+129^2+...+2021^2}{2021}$. [6.6%]

求 $\frac{43^2+86^2+129^2+...+2021^2}{2021}$ 的值。

17) Find the sum of the factors of 202122. [6.7%]

求 202122 的所有因子之和。

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18) Let $A_1, A_2, \ldots, A_m$ and $B_1, B_2, \ldots, B_n$ be primes and $x_1, x_2, \ldots, x_m$ and $y_1, y_2, \ldots, y_n$ be integers. Define [6.8%]

$$\rho\left(\frac{A_1^{x_1}A_2^{x_2}\cdots A_m^{x_m}}{B_1^{y_1}B_2^{y_2}\cdots B_n^{y_n}}\right)=(x_1+x_2+\ldots+x_m)-(y_1+y_2+\ldots+y_n)$$

Suppose $A = 1$, $B = 2$, $C = 3$ and so on, and $ABC = 1 \times 2 \times 3$. Find the value of

$$\rho\left(\frac{MATH}{CONCEPTITION}\right).$$

設 $A_1, A_2, \ldots, A_m$ 和 $B_1, B_2, \ldots, B_n$ 為質數，而 $x_1, x_2, \ldots, x_m$ 和 $y_1, y_2, \ldots, y_n$ 為整數。定義

$$\rho\left(\frac{A_1^{x_1}A_2^{x_2}\cdots A_m^{x_m}}{B_1^{y_1}B_2^{y_2}\cdots B_n^{y_n}}\right)=(x_1+x_2+\ldots+x_m)-(y_1+y_2+\ldots+y_n)$$

假設 $A = 1$，$B = 2$，$C = 3$，如此類推，且 $ABC = 1 \times 2 \times 3$，求

$\rho\left(\frac{MATH}{CONCEPTITION}\right)$ 的值。

19) Define a permutation α of the set A = {0, 1, 2, …, 9} to be [6.9%]

$$\alpha = (\,1\,7\,8\,3\,0\,5\,)(\,4\,9\,6\,2\,)$$

i.e. α (1) = 7, α (7) = 8, …, and α (5) = 1; α (4) = 9, …, and α (2) = 4.

For every element x in A, given that $\alpha^n(x) = \alpha^{n-1}(\alpha\,(x))$ where n is a positive integer. If $\alpha^m(x) = x$, where m is a positive integer, find the minimum value of m.

設 A = {0, 1, 2, …, 9}。定義一個 A 的置換 α 為

$$\alpha = (\,1\,7\,8\,3\,0\,5\,)(\,4\,9\,6\,2\,)$$

即 α (1) = 7，α (7) = 8，…及 α (5) = 1；α (4) = 9，…及 α (2) = 4。

設 x 為 A 的任一元素及 n 為一正整數，已知 $\alpha^n(x) = \alpha^{n-1}(\alpha\,(x))$。如果 $\alpha^m(x) = x$ 而 m 為一正整數，求 m 的最小值。

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20) Find the smallest integer which is greater than $\sqrt{510+2\sqrt{2525}}+\sqrt{510-2\sqrt{2525}}$. [7%]

求大於 $\sqrt{510+2\sqrt{2525}}+\sqrt{510-2\sqrt{2525}}$ 的最小整數。

MATH 數學·思維大激鬥
CONCEPTITION

REG NO		S1
NAME		
GROUP		
SEAT		

ANSWER SHEET

	ANSWER			ANSWER	
1	-144	○	11	7	○
2	$-\frac{1}{40}$	○	12	94	○
3	9y+6	○	13	4	○
4	19	○	14	50	○
5	56	○	15	6	○
6	$48\sqrt{6}$	○	16	32680	○
7	2	○	17	475200	○
8	$33\frac{1}{3}$	○	18	-18	○
9	160	○	19	12	○
10	36	○	20	45	○