MathConception 2019

S3

Question Booklet

問題簿

Name:
姓名：

Reg. No.:
登記編號：

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.
2. Write your name and registration number on the cover of this question booklet.
3. Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.
4. If the information printed on your answer sheet is not correct, please inform the invigilator immediately.
5. Rough-work sheets provided will be collected at the end of the contest but they will not be marked.
6. Diagrams in this question booklet are not necessarily drawn to scale.

限時：1小時
不允許使用計算機。

考生須知：

1. 未宣布開始前，切勿翻閱此問題簿。
2. 請在此問題簿封面的適當位置寫上你的姓名及登記編號。
3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。
4. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。
5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。
6. 此問題簿的附圖不一定依比例繪成。

1) Find the arithmetic mean of the data set 85, 72, 77, 76, 73, 70. [3.0%]

求數據組 85, 72, 77, 76, 73, 70 的算術平均數。

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2) The diameter of a sphere is 10 cm. Find the volume of the sphere in cm^3. [3.1%]
(Give your answer in terms of π.)

一個直徑為 10 厘米的球體的體積是多少立方厘米？(答案以 π 表示。)

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3) Peter lends $10000 to Tom at 4% p.a. compounded quarterly. Find the amount Peter will get after 0.5 years. [3.2%]

小明借了 $10000 給小強，年利率為 4%，每季計算複利息一次。0.5 年後小明所得的本利和是多少？

4) If $\frac{1}{2}(x-1) \leq \frac{1}{3}(x+1)$, where x is an integer, find the largest possible value of x. [3.3%]

若 $\frac{1}{2}(x-1) \leq \frac{1}{3}(x+1)$ 且 x 為整數，求 x 的最大可能值。

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5) How many axis of rotational symmetry does a cube have? [3.4%]

一個立方體有多少條旋轉對稱軸？

6) A cylinder of radius 12 cm contains some water. Three solid spheres of radii 6 cm are put into the cylinder and totally immersed in the water. What is the rise in water level in cm? [3.5%]

一個半徑為 12 厘米的圓柱體容器盛有一些水，若將三個半徑為 6 厘米的實心球體放入圓柱體容器，並完全浸沒於水中，水位會上升多少厘米？

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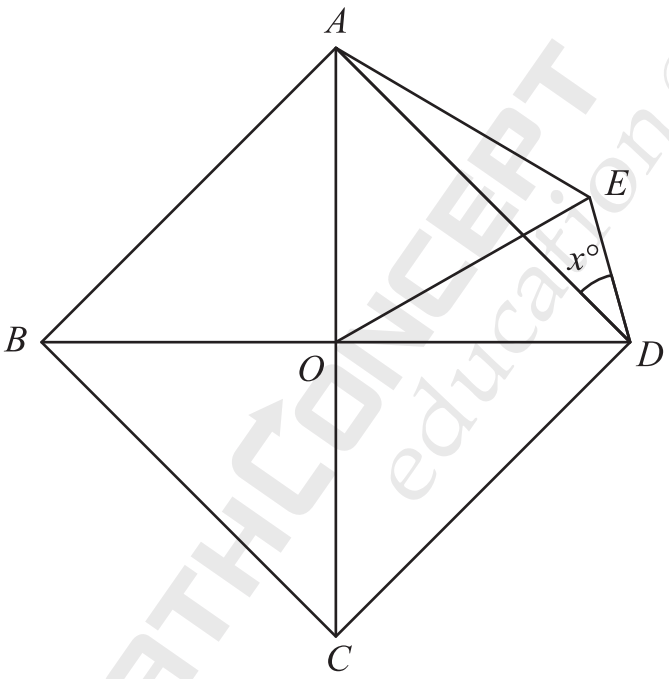
7) Let $\$P$ be the principal, $r\%$ be the annual interest rate and t be the deposit period. If the principal is increased by 30%, the annual interest rate is decreased by its 50% and the deposit period is extended by 30%, the simple interest received will be decreased by $x\%$. Find x. [3.6%]

設 $\$P$ 為本金，$r\%$ 為年利率和 t 為存款期。如果本金增加 30%，年利率減少其原來的 50%，存款期延長 30%，以單利息計算，利息減少了 $x\%$，求 x。

8) The figure shows a square $ABCD$. E is a point outside the square such that ΔAOE is an equilateral triangle. Find x. [3.7%]

圖中 $ABCD$ 是一個正方形。E 是正方形外的一點，使得 ΔAOE 是等邊三角形。求 x。



9) Suppose that $\overline{AB}$ represents a two-digit number. $\overline{AB}$ is less than 54. The sum of A and B is 9. Find the sum of all possible values of B. [4.8%]

$\overline{AB}$ 代表一個小於 54 的兩位數。若 A 和 B 之和是 9，求 B 的所有可能值之和。

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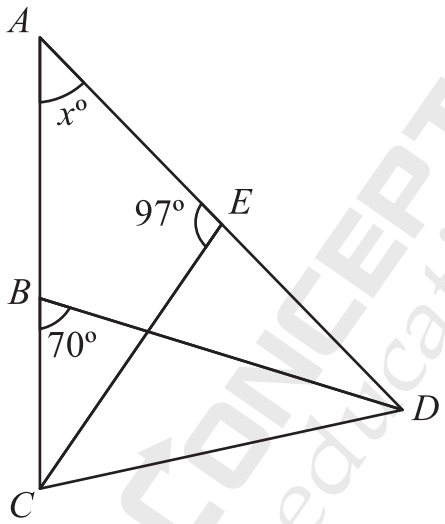
10) The sides of a triangle are 6, 7 and x respectively, where x is an integer. Find the sum of all possible values of x. [4.9%]

已知一個三角形的邊長分別為 6、7 和 x，其中 x 是一個整數。求 x 的所有可能值之和。

11) In the figure, ABC and AED are straight lines. BD and CE are the angle bisectors of $\angle ADC$ and $\angle ACD$ respectively, $\angle DBC = 70°$ and $\angle AEC = 97°$. Find x. [5.1%]

在圖中，ABC 和 AED 是直線。BD 和 CE 分別是$\angle ADC$ 和 $\angle ACD$ 的角平分線，$\angle DBC = 70°$ ，$\angle AEC = 97°$。求 x。



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12) Given that a polyhedron has $(2x-6)$ vertices, $(2x+2)$ edges and (x^2-54) faces, find the number of edges of the polyhedron. [5.2%]

若一個多面體有 $(2x-6)$ 個頂點、$(2x+2)$ 條邊和 (x^2-54) 個面，求多面體的邊數。

13) How many numbers from the first 2019 positive integers are divisible by 4 but not by 6? [6.3%]

首 2019 個正整數中，可被 4 整除但不可被 6 整除的有多少個？

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14) A triangle has side lengths of 14, 48 and 50. If its largest interior angle is $x°$, find $2\sin x°\cos x°$. [6.4%]

某三角形三條邊的長度為 14、48 和 50。若它最大的內角是$x°$，求 $2\sin x°\cos x°$。

15) Find the minimum value of $(x-2)(x-3)(x-4)-(x-6)(x-7)(x-8)$. [6.5%]

求 $(x-2)(x-3)(x-4)-(x-6)(x-7)(x-8)$ 的最小值。

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16) Solve $\dfrac{9x^3+15x^2+9x-1}{3x^2+5x+2}=\dfrac{6x^3+15x^2+17x+1}{2x^2+5x+5}$. [6.6%]

解 $\dfrac{9x^3+15x^2+9x-1}{3x^2+5x+2}=\dfrac{6x^3+15x^2+17x+1}{2x^2+5x+5}$。

17) Find the last two digits of 2020202^{2019}. [6.7%]

求 2020202^{2019} 的最後兩位數字。

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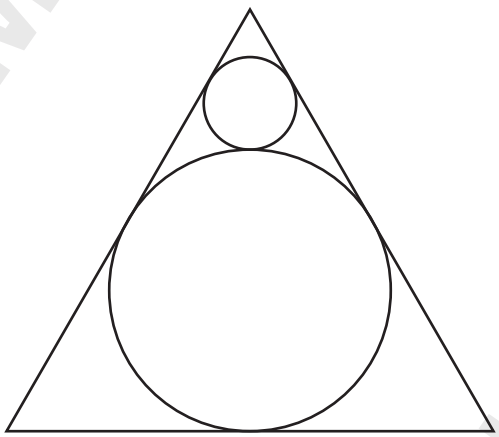
18) In the figure, the larger circle is inscribed in an equilateral triangle. The smaller circle is tangent to the larger circle and two sides of the triangle. If the diameter of the larger circle is $\frac{2\sqrt{\pi}}{\pi}$, find the area of the smaller circle. [6.8%]

在圖中，大圓內接於等邊三角形，小圓與大圓和三角形的邊相切。若大圓的直徑為 $\frac{2\sqrt{\pi}}{\pi}$，求小圓的面積。



19) Given $2(5y - 24x - 6z) = 53 + 16x^2 + 5y^2 + 3z^2$ where x, y and z are real numbers, find $16x + 5y + 3z$. [6.9%]

已知 $2(5y - 24x - 6z) = 53 + 16x^2 + 5y^2 + 3z^2$ 且 x、y 和 z 為實數，求 $16x + 5y + 3z$。

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20) In ΔABC, if $(\sin A + \sin B) : (\sin B + \sin C) : (\sin C + \sin A) = 6 : 4 : 5$, find the value of $\sin^2 B$. [7.0%]

在 ΔABC 中，若 $(\sin A + \sin B) : (\sin B + \sin C) : (\sin C + \sin A) = 6 : 4 : 5$，求 $\sin^2 B$。

End of paper

全卷完

MATH 數學·思維大激鬥
CONCEPTITION

REG NO			S3
NAME			
GROUP			
SEAT			

ANSWER SHEET

	ANSWER			ANSWER	
1	75.5	○	11	42	○
2	$\frac{500\pi}{3}$	○	12	18	○
3	10201	○	13	336	○
4	5	○	14	0	○
5	13	○	15	12	○
6	6	○	16	7	○
7	15.5	○	17	88	○
8	30	○	18	$\frac{1}{9}$	○
9	26	○	19	-25	○
10	77	○	20	$\frac{75}{196}$	○