MathConception 2023

S3

Question Booklet 問題簿

Time: 1 hour
Calculators are NOT permitted.

Instructions:

1. **DO NOT OPEN THIS QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.**
2. **If the information printed on your answer sheet is not correct, please inform the invigilator immediately.**
3. **Please use a pencil and write your answers neatly ONLY on the answer sheet provided. DO NOT write or draw in the circle next to each answer box. No mark will be given if you failed to follow this instruction.**
4. **Unless otherwise specified, all answers must be in exact value and in its simplest form. Writing the units for the answers is NOT necessary.**
5. **Rough-work sheets provided will be collected at the end of the contest but they will not be marked.**
6. **Diagrams in this question booklet are not necessarily drawn to scale.**

限時：1 小時
不允許使用計算機。

比賽須知：

1. 未宣布開始前，切勿翻閱此問題簿。
2. 請核對答題紙上列出的資料是否與你相符。如有問題，請舉手。
3. 所有答案必須寫在答題紙內，並須用鉛筆作答。請勿填寫或畫花題號後方的圓圈，否則該題答案將會作廢。
4. 除非題目特別表明，所有答案均不需填寫單位，但必須以準確數值及最簡方式表示。
5. 比賽完結時監考員會收回桌上的草稿紙，但草稿紙上所書寫的任何文字或圖表將不獲評閱。
6. 此問題簿的附圖不一定依比例繪成。

1) If $\begin{cases} x+y=5 \\ x-y=1 \end{cases}$, find xy. [3%]

若 $\begin{cases} x+y=5 \\ x-y=1 \end{cases}$，求 xy。

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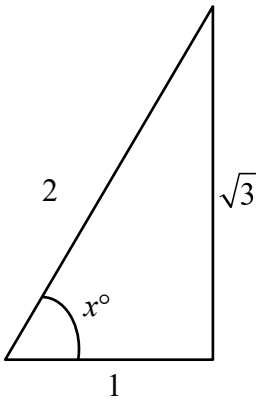
2) Find the arithmetic mean of the data set "35, 45, 55, 65, 75, 85, 95, 105". [3.1%]

求數據組「35、45、55、65、75、85、95、105」的算術平均數。

3) Find the value of x. [3.2%]

求 x 的值。



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4) Simplify $\frac{2x^2+4x+2}{3x^2-3}$, where $x \neq \pm 1$. [3.3%]

化簡 $\frac{2x^2+4x+2}{3x^2-3}$，其中 $x \neq \pm 1$。

5) If a number is first increased by 25%, and then decreased by x%. The value remains unchanged. Find x. [3.4%]

如果一個數字首先增加 25%，然後減少 x%。數值保持不變。求 x。

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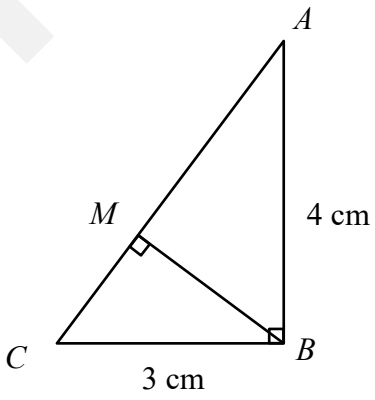
6) The figure shows a right-angled triangle ABC, where $AB = 4$ cm, $BC = 3$ cm, $\angle BMC = 90°$ and $AM = x$ cm. Find x. [3.5%]

圖中，ABC 是一個直角三角形，其中 $AB = 4$ cm，$BC = 3$ cm，$\angle BMC = 90°$ 和 $AM = x$ cm。求 x。



7) Find the sum of all positive integral solution(s) of $\frac{27^x - 3^x}{12^x - 4^x} = 3$. [3.6%]

求 $\frac{27^x - 3^x}{12^x - 4^x} = 3$ 的所有正整數解之和。

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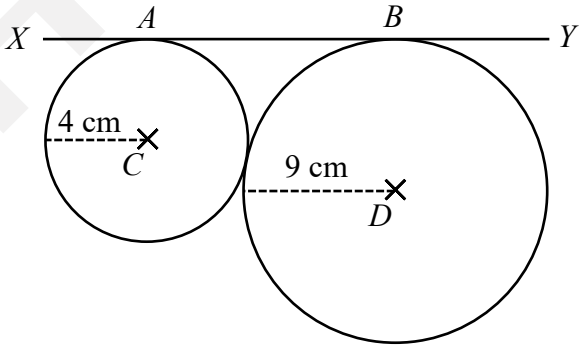
8) In the figure, C and D are the centres of the smaller circle and bigger circle respectively. Their radii are 4 cm and 9 cm respectively. XY is the common tangent to the circles at A and B. If $AB = x$ cm, find x. [3.7%]

圖中，C 和 D 分別是大圓和小圓的圓心，而它們的半徑則分別是 4 cm 和 9 cm。XY 為兩圓的公切線，切點為 A 及 B。若 $AB = x$ cm，求 x。



9) [4.8%]

Given $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$. For example, $|-2| = 2, |0| = 0, |3| = 3$.

If $x^2 - 5|x| + 6 \leq 0$, find the smallest possible value of x.

已知 $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$。例如，$|-2| = 2$，$|0| = 0$，$|3| = 3$。

若 $x^2 - 5|x| + 6 \leq 0$，求 x 的最小可能值。

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10) [4.9%]

Given $C_r^n = \dfrac{n!}{r!(n-r)!}$, where $n! = n(n-1)(n-2)\cdots(2)(1)$.

Find the value of $C_2^{11} + C_3^{11} + C_4^{11} + C_5^{11} + C_6^{11} + C_7^{11} + C_8^{11} + C_9^{11} + C_{10}^{11}$.

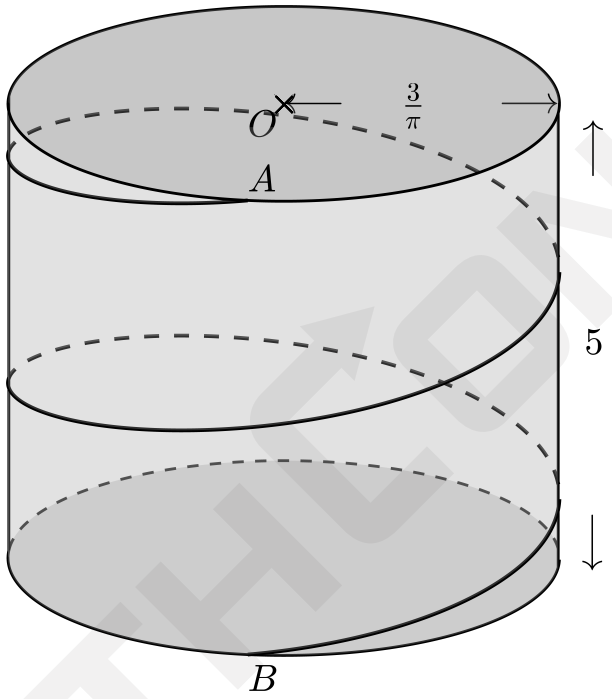
已知 $C_r^n = \dfrac{n!}{r!(n-r)!}$，其中 $n! = n(n-1)(n-2)\cdots(2)(1)$。

求 $C_2^{11} + C_3^{11} + C_4^{11} + C_5^{11} + C_6^{11} + C_7^{11} + C_8^{11} + C_9^{11} + C_{10}^{11}$ 的值。

11) The figure shows a right circular cylinder with base radius $\frac{3}{\pi}$ and height 5. [5.1%]

A and *B* are points on the edge of the two bases and *B* is vertically below *A*. A curve is drawn on the curved surface of the cylinder from *A* to *B* so that it wraps around the cylinder twice. Find the shortest length of the curve *AB*.

圖中為一直立圓柱，底半徑為 $\frac{3}{\pi}$，高為 5。*A* 和 *B* 為兩底邊上的一點，且 *B* 鉛垂於 *A*。在圓柱體曲面上由 *A* 至 *B* 繪畫一條圍繞圓柱 2 圈的曲線，求曲線 *AB* 的最短長度。



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12) If $r = 35 + 12\sqrt{6}$, find $\sqrt{r}$. (Leave your answer in the simplest surd form.) [5.2%]

若 $r = 35 + 12\sqrt{6}$，求 $\sqrt{r}$。（答案以最簡根式表示。）

13) On a rectangular coordinate plane, the graph of equation $|x-10|+|2y|=10$ encloses a region. Find the area of the region. [6.3%]

在直角坐標平面上，方程 $|x-10|+|2y|=10$ 的圖像可圍成一個區域。找出該區域的面積。

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14) Given the general term of a sequence $T_n=\dfrac{2}{4n^2+8n+3}$, [6.4%]

Find the value of $T_1+T_2+T_3+\cdots$.

已知某數列的通項為 $T_n=\dfrac{2}{4n^2+8n+3}$，求 $T_1+T_2+T_3+\cdots$ 的值。

15) 20 identical balls are put 4 distinct boxes, some of which can be empty. Find the number of all different arrangements. [6.5%]

將 20 個相同的球放入 4 個不同的盒子中，有些盒子可以是空的。求所有不同組合的數量。

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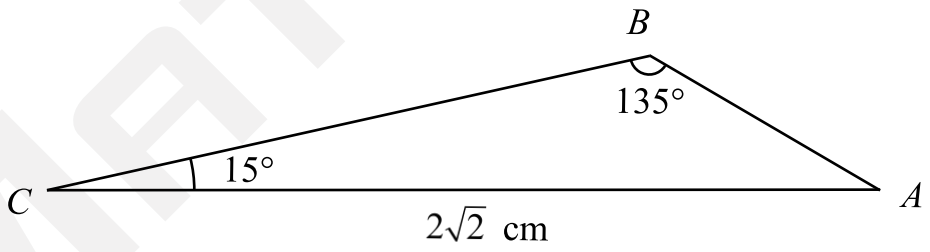
16) In $\triangle ABC$, $\angle ACB = 15°$, $\angle ABC = 135°$, $AC = 2\sqrt{2}$ cm and the area of $\triangle ABC$ is x cm^2. Find x. [6.6%]

在 $\triangle ABC$ 中，$\angle ACB = 15°$，$\angle ABC = 135°$，$AC = 2\sqrt{2}$ cm，$\triangle ABC$ 的面積是 x cm^2。求 x。



17) Solve the equation $\sqrt{x}-\sqrt{x-42291}=111$. [6.7%]

解方程 $\sqrt{x}-\sqrt{x-42291}=111$。

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18) Given that a leap year is the year that is either divisible by 400, or divisible by 4 but not 100. [6.8%]

Today is 19th March 2023, Sunday, which day of the week will 19th March be after 23^{160} years?

已知閏年的年份為 4 的倍數但非 100 的倍數，或年份為 400 的倍數。

今天是 2023 年 3 月 19 日（星期日），23^{160} 年後的 3 月 19 日是星期幾？

19) 7 students write their names on a piece of paper, and put the 7 pieces of paper into a box, then they randomly pick a paper from the box. [6.9%]
Find the probability of all 7 students not getting back their own paper.

7 名學生分別在紙條上寫下自己的名字，再把該 7 張紙條放入一個盒子裡。之後他們在盒子裡隨機挑選一張。
求 7 名學生全部都拿不回自己的紙條的概率。

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20) Find the Highest Common Factor of $2^{2024}-1$ and $2^{2008}-1$. [7%]

求 $2^{2024}-1$ 和 $2^{2008}-1$的最大公因數。

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MATH 數學·思維大激鬥 CONCEPTITION

REG NO			S3
NAME			
GRADE			
SEAT			

ANSWER SHEET

○ IDCHECK ○ ABSENT

	ANSWER			ANSWER	
1	6		11	13	
2	70		12	$2\sqrt{2}+3\sqrt{3}$	
3	60		13	100	
4	$\frac{2(x+1)}{3(x-1)}$		14	$\frac{1}{3}$	
5	20		15	1771	
6	$\frac{16}{5}$ or $3\frac{1}{5}$ or 3.2		16	$\sqrt{3}-1$	
7	1		17	60516	
8	12		18	Tuesday	
9	–3		19	$\frac{103}{280}$	
10	2035		20	255	

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